Correlation and Causality

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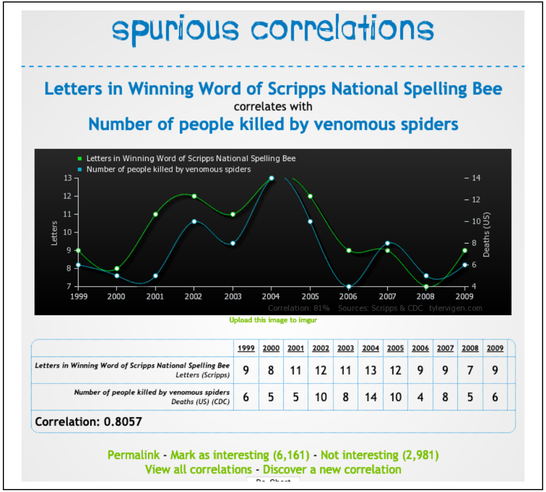
Why causality matters

Because correlation is a proxy.



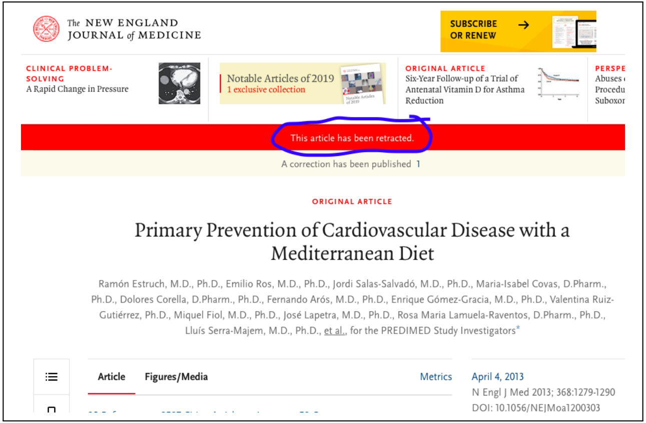
	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Letters in Winning Word of Scripps National Spelling Bee *Letters (Scripps)*	9	8	11	12	11	13	12	9	9	7	9
Number of people killed by venomous spiders *Deaths (US) (CDC)*	6	5	5	10	8	14	10	4	8	5	6

Correlation: 0.8057

[Vig]

Why causality matters

Because A / B testing is not always possible.



[ERSS+13]

Simpson's paradox: cautionary tales

Simpson's paradox: a phenomenon in probability and statistics in which a trend appears disappears or reverses depending on grouping of data. [Wik], [PGJ16]

Example: University of California, Berkeley 1973 admission figures

	Men		Women	
	Applicants	Admitted	Applicants	Admitted
Total	8442	**44%**	4321	35%

[FPP98]

Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	***825***	62%	108	**82%**
B	***560***	63%	25	**68%**
C	325	**37%**	***593***	34%
D	417	33%	375	**35%**
E	191	**28%**	***393***	24%
F	373	6%	341	**7%**

[BHO75]

A brief, biased history of causality

- Aristotle, 384 - 322 BC
- Isaac Newton, 1643 - 1727 AD
- David Hume, 1711 - 1776 AD
- Francis Galton, 1822 - 1900 AD, Karl Pearson, 1857 - 1936 AD
- Judea Pearl, b. 1936 AD

Counterfactuals and causality

Ideal: Intervention + Multiverse → Causality

Examples:

- Medical treatment (e.g. kidney stone treatment)
- Social outomes (e.g. university admissions)
- Business outcomes (e.g. click-through rate, hit rate)

In-practice:

- Correlation: approximate multiverse by comparing intervention at t to result at $t - 1$
- Random population: approximate multiverse by splitting sample well
- A / B testing: random populations A / B + intervention in one

Counterfactual example: hit rate for insurance

Variables:

- producttype: Client line of business
- days: Number of days to generate quote
- rating: Binary indication of client risk
- hit: Binary, 1 for success (binding the quote), 0 for failure

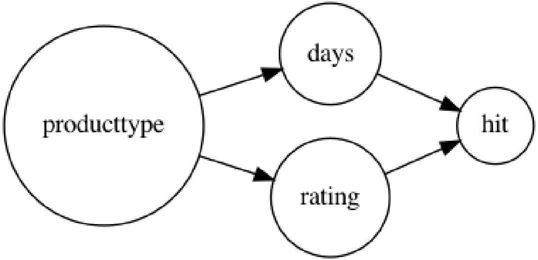
Fake data:

product_type	days	rating	hit
property	3	1	0
liability	1	0	0
financial	0	1	0
liability	3	0	0
liability	0	0	1

Counterfactual example: hit rate for insurance

Variables:

- producttype: Client line of business
- days: Number of days to generate quote
- rating: Binary indication of client risk
- hit: Binary, 1 for success (binding the quote), 0 for failure



Non-counterfactual approach: condition and query

Goal: estimate effect of days on hit.

Calculate

- $P(\mathrm{hit} = 1 | \mathrm{days} = 0) - P(\mathrm{hit} = 1 | \mathrm{days} = 1)$,
- $P(\mathrm{hit} = 1 | \mathrm{days} = 1) - P(\mathrm{hit} = 1 | \mathrm{days} = 2)$,
- $\ldots$

From exercise Jupyter notebook:

	hit
days	
0	0.532706
1	0.442064
2	0.330519
3	0.174006

The Structural Causal Model

The definitions in following slides are from [Pea07], [PGJ16].

Definition

A *structural causal model* M consists of two sets of variables U, V and a set of functions F, where

- U are considered *exogenous*, or background variables,
- V are the *causal* variables, i.e. that can be manipulated, and
- F are the functions that represent the process of assigning values to elements of V based on other values in U, V, e.g. $v_i = f(u, v)$.

We denote by G the graph induced on U, V by the functions F, and call it the *causal graph* of (U, V, F).

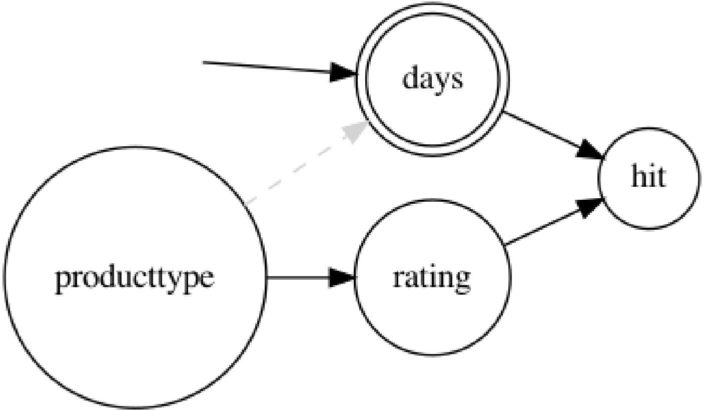
Hit rate example: $U = \{\text{producttype}, \text{rating}\}$, $V = \{\text{days}, \text{hit}\}$, $F \leftrightarrow$ sample from conditional probabilty tables in directed graphical model.

Formalizing interventions: the intuition of "do"

For business application, quantity of interest is not $P(\mathrm{hit} = 1 | \mathrm{days} = d)$, but intervention

$$P(\mathrm{hit} = 1 | \mathrm{do}(\mathrm{days} = d))$$

.

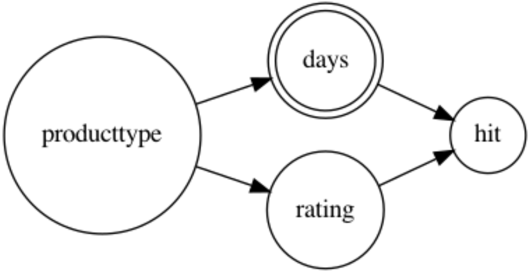


Formalizing interventions: the intuition of “do”

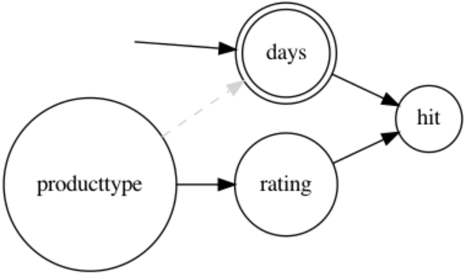
For business application, quantity of interest is effect of intervention / counterfactual

Not $P(\text{hit} = 1 | \text{days} = d)$ but $P(\text{hit} = 1 | \text{do}(\text{days} = d))$

$G =$

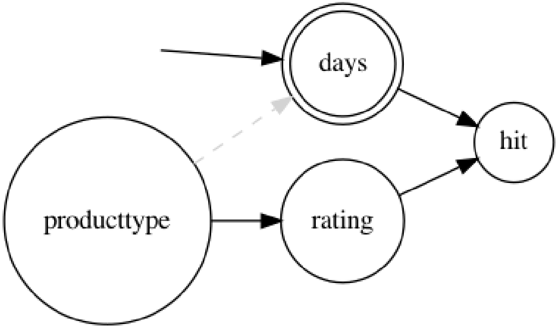


$G' = G_{\underline{\text{days}}} =$



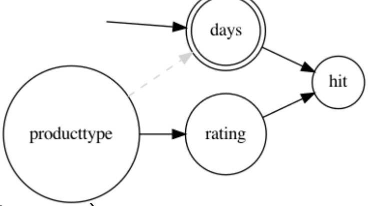
Formalizing interventions: the intuition of "do"

First, find quantities unchanged between G and $G' = G_{\underline{\text{days}}}$



$$
\begin{aligned}
&P_{G'}(\text{producttype} = p, \text{rating} = r) \\
&\qquad = P_G(\text{producttype} = p, \text{rating} = r) \qquad (1)\\
&P_{G'}(\text{hit} = 1 | \text{producttype} = p, \text{rating} = r) \\
&\qquad = P_G(\text{hit} = 1 | \text{producttype} = p, \text{rating} = r) \qquad (2)
\end{aligned}
$$

Formalizing interventions: the intuition of "do"



$$
\begin{aligned}
&P(\text{hit} = 1 | \text{do}(\text{days}) = d) \\
&\quad = P_{G'}(\text{hit} = 1 | \text{days} = d), \text{ by definition} \\
&\quad = \sum_{p,r} P_{G'}(\text{hit} = 1 | \text{days} = d, \text{producttype} = p, \text{rating} = r) \\
&\qquad\qquad P_{G'}(\text{producttype} = p, \text{rating} = r | \text{days} = d), \text{ by total probability} \\
&\quad = \sum_{p,r} P_{G'}(\text{hit} = 1 | \text{days} = d, \text{producttype} = p, \text{rating} = r) \\
&\qquad\qquad P_{G'}(\text{producttype} = p, \text{rating} = r), \text{ by substitution} \\
&\quad = \sum_{p,r} P_{G}(\text{hit} = 1 | \text{days} = d, \text{producttype} = p, \text{rating} = r) \\
&\qquad\qquad P_{G}(\text{producttype} = p, \text{rating} = r), \text{ our } \textit{adjustment} \text{ formula}
\end{aligned}
$$

References: [PGJ16], [Pro]

Causal hit rate

Typical quantity of interest: *average treatment effect* or *ATE*

$P(\mathrm{hit} = 1 | \mathrm{days} = d)$

	hit
days	
0	0.532706
1	0.442064
2	0.330519
3	0.174006

$P(\mathrm{hit} = 1 | \mathrm{do}(\mathrm{days} = d))$

	prob
days	
0	0.565343
1	0.397330
2	0.240322
3	0.215639

Causal hit rate, II

Compute relative average treatment effect for different values of days:

$$\mathrm{relative\text{-}ate}_G = \frac{P_G(\mathrm{hit} = 1 | \mathrm{days} = d) - P_G(\mathrm{hit} = 1 | \mathrm{days} = d+1)}{P_G(\mathrm{hit} = 1 | \mathrm{days} = d)}$$

$$\begin{aligned}\mathrm{relative\text{-}ate}_{G'} &= \frac{P_G(\mathrm{hit} = 1 | \mathrm{do}(\mathrm{days} = d)) - P_G(\mathrm{hit} = 1 | \mathrm{do}(\mathrm{days} = d+1))}{P_G(\mathrm{hit} = 1 | \mathrm{do}(\mathrm{days} = d))} \\ &= \frac{P_{G'}(\mathrm{hit} = 1 | \mathrm{days} = d) - P_{G'}(\mathrm{hit} = 1 | \mathrm{days} = d+1)}{P_{G'}(\mathrm{hit} = 1 | \mathrm{days} = d)}\end{aligned}$$

from-d	to-d	ate-given	ate-do
0	1	0.170153	0.297187
1	2	0.252329	0.395158
2	3	0.473538	0.102707

Judea Pearl's Rules of Causality

Let X, Y , Z and W be arbitrary disjoint sets of nodes in a DAG G. Let $G_{\underline{X}}$ be the graph obtained by removing all arrows pointing into (nodes of) X. Denote by $G_{\overline{X}}$ the graph obtained by removing all arrows pointing out of X. If, e.g. we remove arrows pointing out of X and into Z, we the resulting graph is denoted by $G_{\underline{X}\overline{Z}}$
Rule 1: Insertion / deletion of observations

$$P(y|\mathrm{do}(x), z, w) = P(y|\mathrm{do}(x), w) \text{ if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}}}$$

Rule 2: Action / observation exchange

$$P(y|\mathrm{do}(x), \mathrm{do}(z), w) = P(y|\mathrm{do}(x), z, w) \text{ if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}\underline{Z}}}$$

Rule 3: Insertion / deletion of actions

$$P(y|\mathrm{do}(x), \mathrm{do}(z), w) = P(y|\mathrm{do}(x), w) \text{ if } (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}\overline{Z(W)}}},$$

where $Z(W)$ is the set of Z-nodes that are not ancestors of any W-node in $G_{\underline{X}}$.

Special cases of the causal rules

By judicious setting of sets of nodes to be empty, we obtain some useful corollaries of the causal rules.

Rule 1': Insertion / deletion of observations, with $W = \emptyset$

$$P(y|\mathrm{do}(x), z) = P(y|\mathrm{do}(x)) \text{ if } (Y \perp\!\!\!\perp Z|X)_{G_{\overline{X}}}$$

Rule 2': Action / observation exchange, with $X = \emptyset$

$$P(y|\mathrm{do}(z), w) = P(y|z, w) \text{ if } (Y \perp\!\!\!\perp Z|W)_{G_{\underline{Z}}}$$

Rule 3': Insertion / deletion of actions, with $X, W = \emptyset$

$$P(y|\mathrm{do}(z)) = P(y) \text{ if } (Y \perp\!\!\!\perp Z)_{G_{\overline{Z}}}$$

Special cases of the causal rules

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Rule 3': Insertion / deletion of actions, with $X, W = \emptyset$

$$P(y|\mathrm{do}(z)) = P(y) \text{ if } (Y \perp\!\!\!\perp Z)_{G_{\overline{Z}}}$$

$\Longrightarrow$ d-separation + causal rules = *adjustment formulas*: do queries as normal queries.

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